(S.02)

Two Fold eqn. of Two line not passing

through the origin

$$ax^2 + by^2 + 2hxy + 2fx + 2gy + c = 0$$

$$\Delta = \begin{vmatrix} a & h & f \\ h & b & g \\ f & g & c \end{vmatrix} = 0$$

equation of Bisectors for Two eqn not passing through (0,0)

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

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EX :-

$(a+b+c)^2 = a^2+b^2+c^2+2[ab+ac+bc]$

$y^2 - 6xy + 8x^2 - 2x - y - 6 = 0$

prove that this represent Two Lines, find them, Angle, Pointe of intersection

$$\Delta = \begin{vmatrix} 8 & 3 & -1 \\ 3 & 1 & -\frac{1}{2} \\ -1 & -\frac{1}{2} & 6 \end{vmatrix} = 8(6-\tfrac{1}{4}) - 3(18-\tfrac{1}{2}) - 1(-\tfrac{3}{2}+1) = 0 \quad \#$$

$8x^2 - 6xy + y^2 = 0$

$(x-y)(4x-y) = 0$

$$\frac{y}{x} = \frac{-h \pm \sqrt{h^2-ab}}{b}$$

$y = 4x \quad , \quad y = 2x$

$y - 4x + \alpha = 0$

$y - 2x + \beta = 0$

$(y - 4x + \alpha)(y - 2x + \beta) = 0$

cf x: $[-4\beta - 2\alpha] = -2$

Cof y: $(\beta + \alpha) = -1$

Free Term: $\alpha\beta = 6$

$2\beta + 2\alpha = -2$

$-4\beta - 2\alpha = -2$

$-2\beta = -4$

$\beta = 2 \Rightarrow \alpha = 3$

$y - 4x + 3 = 0 \quad , \quad y - 2x + 2 = 0 \quad \#$

$$\tan\theta = \frac{2\sqrt{9-8}}{1+8} = \frac{2}{9}$$

$\theta = 12.5^\circ \quad \#$

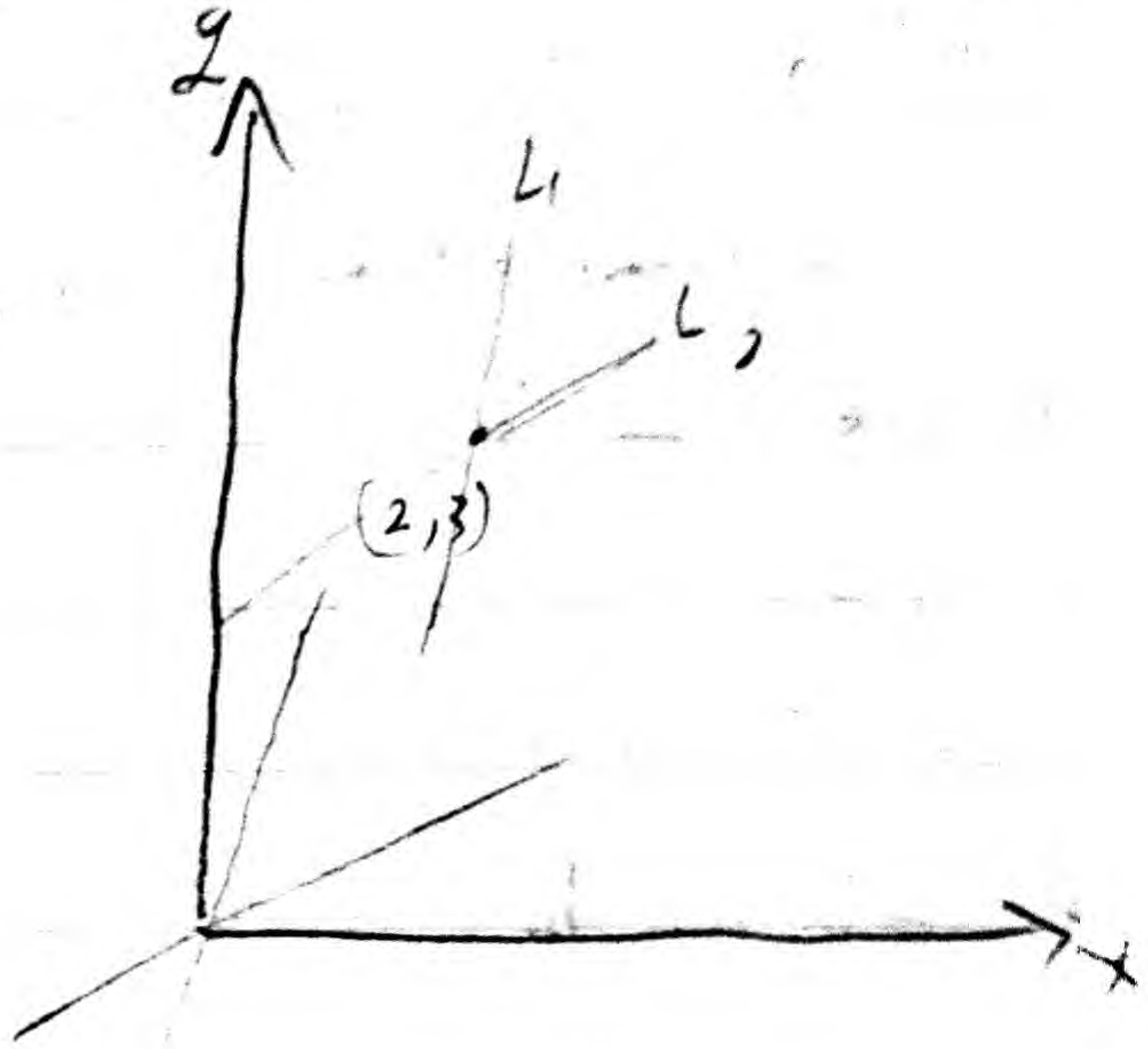
$$\frac{y - 4x + 3}{\sqrt{1+16}} = \frac{y - 2x + 2}{\sqrt{1+4}}$$

$$\frac{(y - 4x + 3)^2}{17} = \frac{(y - 2x + 2)^2}{5}$$

تابع وجمع

EX: Find Two Pair eqn of Two Lines that passing through (2, 3) and parallel to the lines $2X^2 - 5Xy + y^2 = 0$

Then find the Bisectors



$h = \frac{-5}{2}$, $a = 2$, $b = 1$

$$\frac{y}{x} = \frac{-h \pm \sqrt{h^2 - ab}}{b}$$

$\frac{y}{x} = 0.438$

$\frac{y}{x} = 4.563$

L_1: $y - .438X + \alpha = 0$

L_2: $y - 4.563X + \beta = 0$

(2, 3)

$3 - (2)(.438) + \alpha = 0 \Rightarrow \alpha = -2.124$

$3 - (2)(4.563) + \beta = 0 \Rightarrow \beta = 6.25$

CoF X [✓]   CoF y [✓]   Freeterm [✓]

$2X^2 - 5Xy + y^2 + 2fX + 2gy + c = 0$

CoF X: $[-0.43 * \beta - 4.563\alpha]$

CoF y: $[\beta + \alpha]$

Freeterm: $\alpha\beta$

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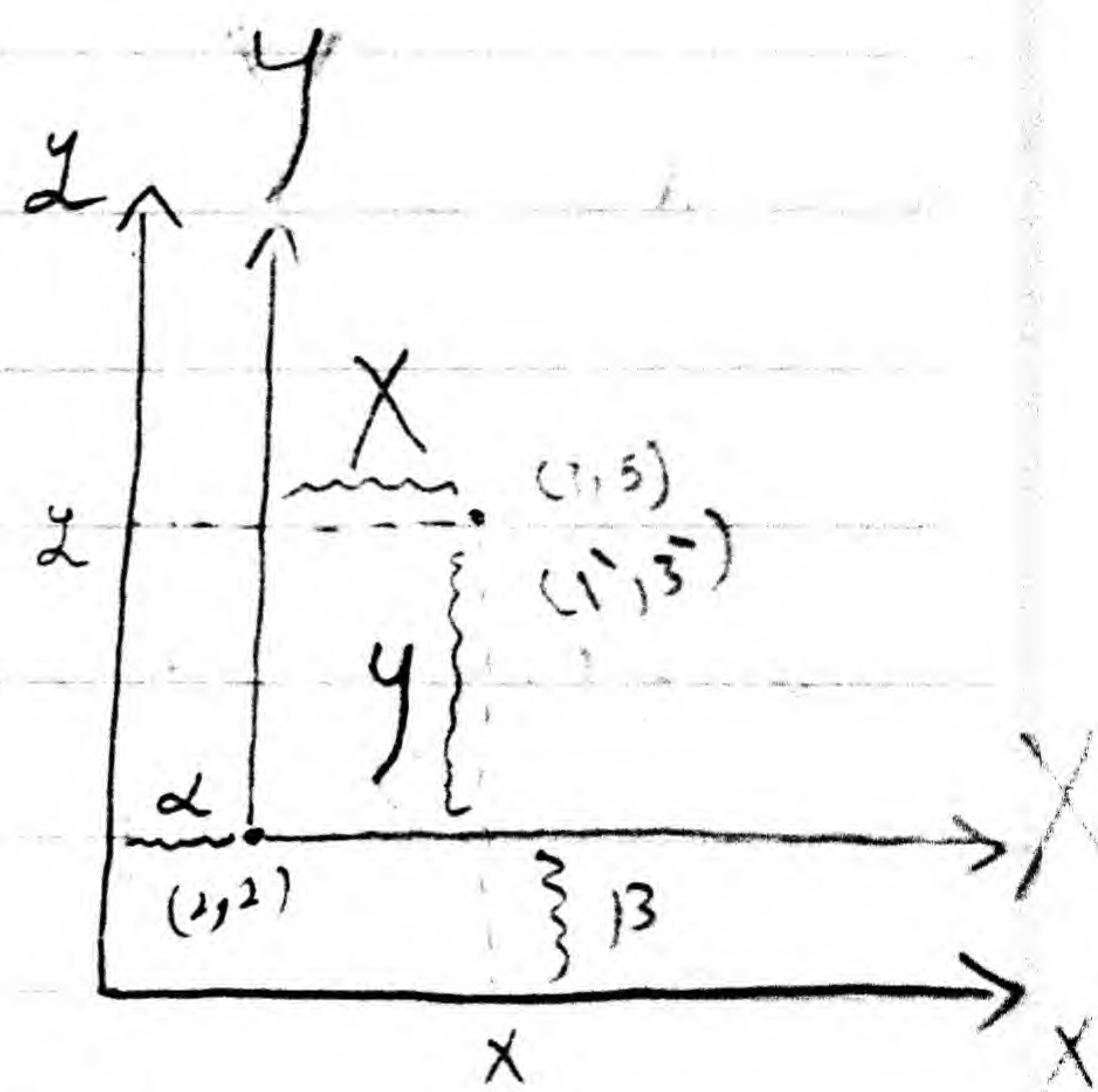
$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \Rightarrow \theta = 53.26^\circ$$

$$\frac{y - .438X - 2.124}{\sqrt{1^2 + (.438)^2}} = \frac{y - 4.563X + 6.25}{\pm\sqrt{1^2 + (4.563)^2}}$$

Translation of axes :-

$$x = X + \alpha$$
$$y = Y + \beta$$



$(\alpha, 0) \Rightarrow x = X + \alpha \ , \ y = Y$

$(0, \beta) \Rightarrow x = X \ , \ y = Y + \beta$

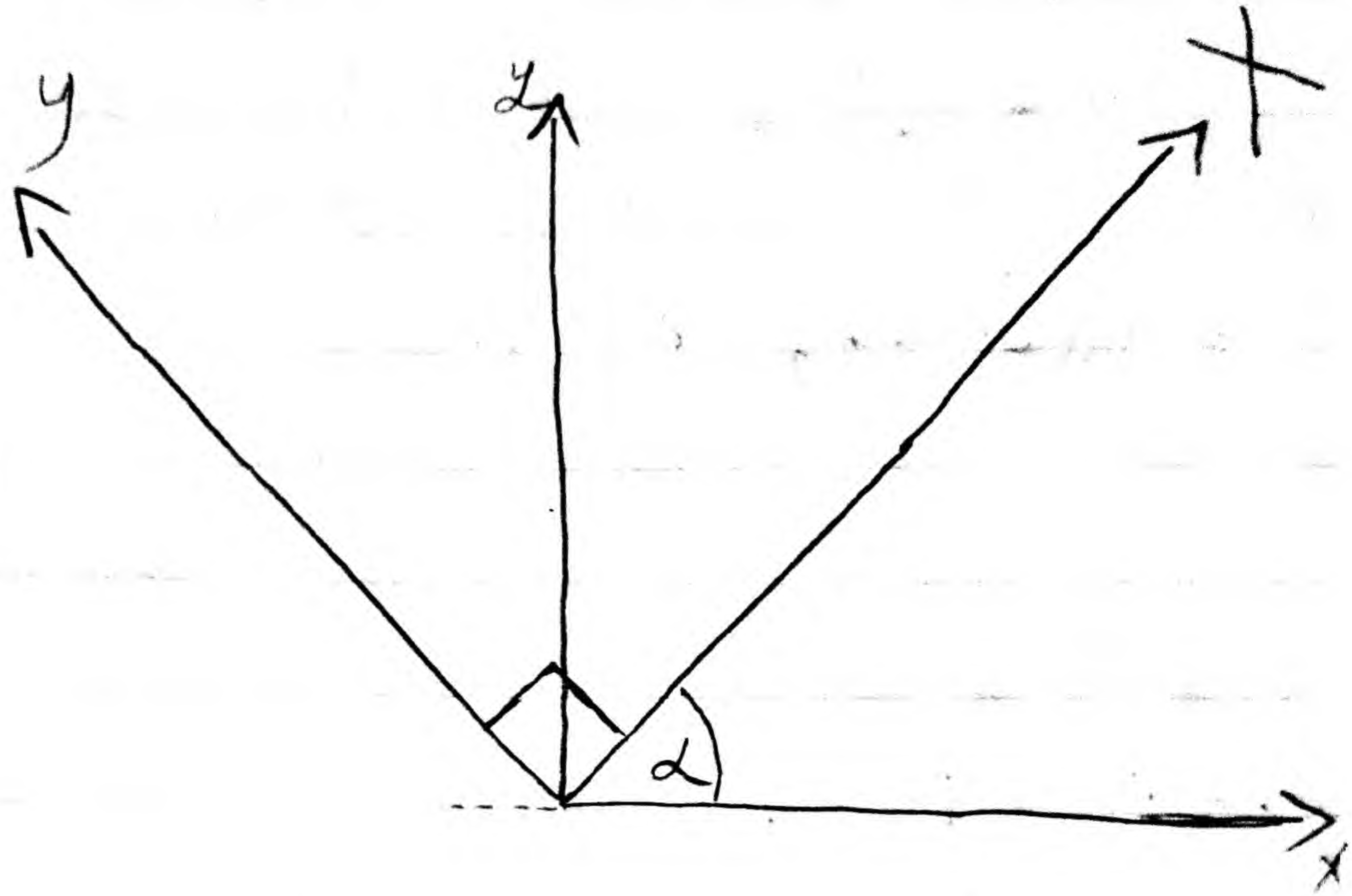
F تابع $\longrightarrow$ ① $\frac{dF}{x} = 0$, ② $\frac{dF}{y} = 0$

(1, 2) Solve $(\alpha, \beta) \longrightarrow$ eliminate X, Y termer

Rotation of axes:-

x, y elimination

$$\tan\theta = \frac{2h}{a-b}$$



	x	y
X	$\cos\theta$	$\sin\theta$
Y	$-\sin\theta$	$\cos\theta$

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EX: eliminate The first degree terms, then the term xy:-

$$2x^2+5y^2+4xy-4x-22y+7=0$$

$$\frac{\partial f}{\partial x}=4x+0+4y-4=0$$

Solve

$$\frac{\partial f}{\partial y}=10y+4x-22=0$$

$$(\alpha,\beta)=(-2,3)$$

$$\boxed{x=X-2}$$
$$\boxed{y=Y+3}$$

$$2(X-2)^2+5(Y+3)^2+4(X-2)(Y+3)-4(X-2)-22(Y+3)+7=0$$

$$2X^2+5Y^2+4XY+C=0$$

$$C=f(\alpha,\beta)=f(-2,3)=-22$$

$$2X^2+5Y^2+4XY-22=0 \quad *$$

$$\tan(2\theta)=\frac{4}{2-5}=\frac{-4}{3}$$

$$\theta=26.56^\circ$$

$$X=X\cos\theta-Y\sin\theta$$
$$Y=X\sin\theta+Y\cos\theta$$
$$X=X\cos26.56-Y\sin26.56$$
$$Y=X\sin26.56+Y\cos26.56$$

$$2(X\cos\theta-Y\sin\theta)+5(X\sin\theta+Y\cos\theta)$$
$$+4(X\cos\theta-Y\sin\theta)(X\sin\theta+Y\cos\theta)$$
$$-22=0$$